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ABSTRACT

Instructional techniques are outlined, as utilized in a remedial algebra course offered at the Community College of Allegheny County. The paper first provides background information concerning the college, the characteristics of its students, the content of the remedial course, and the testing procedures used to place students in the course. The bulk of the paper then describes and provides illustrative examples of several factors that have contributed to course success: (1) careful verification by the instructor that the students have been properly placed; (2) utilization of a lecture/discussion format to involve students in identifying the meaning of a problem, viewing the problem as a whole, employing a flexible approach to its solution, and developing solution strategies; (3) provision for a cumulative review of a student's mastery; (4) student contributions to the solution of a problem; (5) use of surprise quizzes; (6) periodic calculation of students' grade point average as a means of providing feedback; (7) use of procedures which allow students to review old exams, provide feedback after the exam, and allow for instructor comments on test papers; and (8) maintenance of faculty self-esteem through collegial and administrative support. The report concludes with a rationale used in the course to explain the importance of studying mathematics. (JP)

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FUNDAMENTAL ALGEBRA:

INCREASING MASTERY AND RETENTION IN COLLEGE

by

Lois R. Allen

JC 810 356

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March 20, 1981

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About Allegheny Campus of Community College of Allegheny County, 808 Ridge Avenue, North Side, Pittsburgh, Pa. 15212 ;

Founded in 1966, Allegheny Campus is part of the 4-campus system of Community College of Allegheny County. At Allegheny Campus presently are 3,085 students registered full time and 915 registered part time in day classes, with many other evening students. The largest division is the Business/Math/Engineering Division, the largest number of students majoring in business.

The average age of the student body is 27, but the age in my classes is much lower. Most of my students have graduated from high school in the past several years, but some have been out of school for 10-15-20 years, some have GED's, some are veterans, some students are being sent to college by the Bureau of Vocational Rehabilitation (BVR); some have me sign their attendance forms for C.E.T.A. or Job Power programs, which pay them while they attend college. Many of my students work part or full time or have responsibilities as parents or heads of households. In previous semesters my students have included blind or disabled students. Most of the students at Allegheny Campus have received some form of financial aid. About 50% transfer to four-year institutions while the others go directly into careers.

Our Mathematics Department is served by 15 full-time instructors and 38 part-time instructors. Most of the permanent faculty has taught here 10 years or more. A full load is 15 hours/week, and we may opt for one teaching overage in the day or evening at extra pay. The department consensus is that, in the future, they would like to have a Mathematics lab to supplement their teaching. Most Mathematics courses are 4 credits, 4 days/week, except for two courses:

Computation Fundamentals (4 hours, 0 credit)

and Fundamentals of Algebra (4 hours, 0 credit)

The course I am discussing today, Fundamentals of Algebra, meets 4 days per week, or 2 double classes per week; we are teaching 37 sections of it this semester; it is taught this semester by 12 of the full-time professors and 9 part-timers. Sections are offered from 8:00 A.M. to 8:00 P.M. and on Saturdays. In 15 weeks (+ 1 Exam week) we cover Signed Numbers, Sets, Polynomials, Linear Equations, Special Products and Factors, Fractions and Introduction to the Cartesian Coordinate System and Radicals. The syllabus, which all instructors follow, was shortened several years ago, leaving much of the work on radicals to the next courses, Intermediate Algebra, Technical Mathematics, or Mathematics for Health Sciences. Depending upon their majors, students who pass Fundamentals of Algebra go on for credit to a credit course, such as Intermediate Algebra, Technical Mathematics, Mathematics for Health Sciences, Modern Mathematics, Basic Applied Mathematics, Geometry or SI Metric System. Although only three credits of Mathematics is required for graduation, most students find their majors require additional Mathematics courses beyond these courses.

All entering students, except transfer students, are required to take Co-op Placement Tests in Mathematics, English, and Reading. The Admissions Office gives students a choice of taking the Arithmetic, Algebra, Algebra 2, or Trigonometry test. The department established that a student who scores between 15 and 22 correct on the 40-question Co-op Test for Algebra, be placed in Fundamentals of Algebra. Several years ago the College agreed with us that waivers for developmental courses had not worked. No waivers of course placement in Fundamentals of Algebra are now permitted, but a student may re-take the placement test if he/she

feels there has been a mistake. Several years ago I made a study comparing placement scores and grades in initial Mathematics courses. Overall, there was no correlation; however, I did find that, when a student was given the appropriate Mathematics placement test and when a student was placed in the recommended course, according to Department guidelines, there was a correlation. Our Department is now in the process of making up and testing our own placement tests, for future use.

The students take lab courses on their computer, science, engineering and other courses. But I believe most of our Department feels that the lecture classes in Mathematics offer the student opportunities not usually emphasized as much in the lab: group interaction, methods of setting down and approaching problems, learning from others, explaining to others, the strong influence of a teacher. Many of our students have been poorly motivated, were underachievers, are inclined to give up easily, and they need the structured-type situation, it appears.

With this background in mind, I present to you some approaches and techniques I have used in my developmental classes in Fundamentals of Algebra. While I now use them, I recognize they probably would not fit into every instructor's situation. I, myself, am continually trying to improve on this and be flexible to the needs of the students.

A 37-year-old student can not be considered for acceptance into the Nursing Program until she gets past developmental courses, like Fundamentals of Algebra. An engineering, business, or computer science, or biology student has to pass Fundamentals of Algebra in order to take more advanced courses in his or her major. Most of the students took at least a year of Algebra before (180 classes) and either did well and haven't used it or have forgotten it or didn't learn it well enough; thus, the college-level instructor needs to try an approach that may be different but provides whatever it takes to help the student to learn the basis of Algebra in a short 15-week, 60-class session.

Today I plan to discuss what I have found seems to work in my classes, to get across Fundamentals of Algebra:

1. Correct placement
2. Lecture - work session classes
3. Emphasis on
   - Meaning
   - The whole problem
   - Flexibility
   - Strategy
4. Mastery
5. Cumulative reviewing
6. Student involvement and participation
7. Outside help
8. Check tests
9. Cumulative grades
10. Tests and Exams - before, during, and after
11. Faculty self-esteem, appreciation, and support.

We attempt all of these with the hope that the student will be able to make a successful transition to the next Mathematics course.

A. To Aid Understanding and Mastery of beginning Algebra:

1. Placement

The first day of class our instructors ask the students about their Mathematics backgrounds. In between classes, we check their placement records to determine if the students are correctly placed. If we find they are in the wrong course, because of placement or because they failed the computations course, we try to advise them right away to make the course change. Once a student has attended a class for several sessions, we find it hard to get him/her to drop down to the lower course.

During the first three weeks we try to give at least one test, so that the student can make an intelligent decision about making a vertical change, permitted through Week 3.

After Week 3 the student may withdraw officially through Week 10. We make every effort those first 3 weeks to see that the student is in the right course.

2. Lecture - Work-session Classes

For the reasons mentioned earlier, we use the lecture method. But one of the main objectives is to keep the student <u>involved</u> in the course - to make a non-participant into an active participant. I work the problem, step by step, at the board, while they work it at their seat. After awhile, I will ask a student if he/she can tell us the next step. They have the opportunity at each step to stop and ask a question.

We try to emphasize that the <u>key</u> to working a problem is: what does the problem <u>mean</u>?

When I feel it will not threaten the students, I ask them to

explain what they did. People with college courses will probably be called upon to explain things to their co-workers in their jobs.

We try to stress also that it helps to see the problem as a whole and to be flexible in looking at and working Mathematics problems.

Take the simple problem: Subtract - 12 from +7.

First, after establishing on the number line the difference between any addition problem and any subtraction problem, we look at this problem in several forms, as it might be presented in a problem:

(a) Subtract -12 from +7

(b) From +7 subtract -12

(stressing that it is -12 which is being subtracted either way)

(c) Vertically

(e) Converted to addition

Subtract	→	Add
+7		+7
⊖12	→	⊕12
		+19

Concluding:

Subtracting a -12 gives the same answer as to add a +12, or you can change any subtraction problem to an addition problem by changing the sign of the Subtrahend and then following the addition rule.

Students are able to explain and to convert these subtraction problems to equivalent addition problems.

(d) Horizontally

$(+7) \underset{\text{Subtract}}{\longrightarrow} (-12)$

$= +7 + 12 = \boxed{+19}$

We continually stress that while certain rules are necessary, if students go back to what the problem means or to certain basic properties, like the distributive property, they will probably be

able to work the problem. Students sometimes tend to think of Mathematics as rigid in approach. It is precise, but Mathematics allows for a lot of flexibility and versatility of approach, once certain laws or properties are observed. I stress that there is no one right way. We stress that Mathematics is said to help one to separate the important from the unimportant.

Still, Mathematics is precise. We continually go back to problems like these 2 multiplication problems

(a) $5X\cdot(3X-Y)$ vs. (b) $5X(3X)(-Y)$,

asking, "What does each problem mean?"

Developing Strategy is one of the important outcomes of good mathematical learning. With so many steps and operations to choose from, the student is encouraged to look at a problem as

(a) I am here (the problem)

and

(b) I want to get there (the solution)

asking "How do I get there?"

Analogies are often discouraged, but I compare problem-solving to throwing a ball. When a student throws a ball, he/she doesn't look at the ball. He/she keeps his/her eye on where the ball is being thrown.

So, in solving a simple linear equation such as

$$3(3X-4) = 2(X+5) -12$$

we bring out the strategy in solving:

Isolate the variable.

There are many next steps the student may take, but the final goal is to end up with an equivalent equation with X on one side

and its value on the other. Of course, we try to bring in sound mathematical theory, also.

Deciding which of several correct approaches, builds confidence in the student.

When the solution is $X = \frac{10}{7}$, many students groan at fractional answers, wondering if they made a mistake. These solutions, we encourage them, are harder to check, but they represent more of an accomplishment when checked, for the check required more perseverance and confidence than did the solution. But what a satisfaction it is to <u>know</u> that your answer is correct!

For a smoother transition to the next course, we increasingly add more sophisticated approaches to our problems.

4 - 5. Mastery and Cumulative Reviewing

While understanding is necessary, Mastery is at least equally important.

I found that, after completing signed numbers and also polynomials, students become confused when all the basic operations appeared together in tests.

To pull together these 4 basic operations, I conclude the unit with review, performing all 4 operations on the same two numbers. For example: Given +12
- 4

I ask them to give: (a) Sum _____
(b) Difference _____
(c) Product _____,
and (d) Quotient _____.

This seemed to pull together the 4 operations, especially when the students are asked to work them without pencils when several such problems are written on the board.

I tried the same procedure to review the chapter on polynomials.

For example. Given: (1) $-12x^3$ $+4x^3$ (Like Terms) (2) $15a^2$ $+5a$ (Unlike Terms)

and asking them to find the sums, differences, products, and quotients.

Using this review seems to have improved the students' mastery of the 4 basic operations on tests. For several years I have included several such problems on the tests on signed numbers and polynomials, but I am not sure it is good. I think it may confuse the student in a testing situation -- going from adding to subtracting, etc., and it may be better just to do all addition problems together. But, for review, I believe it helps, this cumulative reviewing.

Take these 3 Factoring by Grouping problems:

(1) $6xy - 3y^2 - 14x + 7y$ (2:1; +,−) (2:1; −,+)

(2) $3ab + 4ax + 6bc - 8cx$ (3:4; +,+) (3:4; +,−)

(3) $27x^2 + 12x - 8y - 18xy$ (9:4; +,+) (4:9; −,−)

Students can be shown that, for a "2 and 2" factoring by grouping, two <u>possibilities</u> must exist in the group:

(a) The same Ratio of Coefficients

and

(b) The same Sign Sequence.

They can be shown that, while some expressions can be worked around to that factoring type, as in #1 and #3, there is no way that in #2, two positives can ever become a positive and a negative.

6. Student Involvement and Participation

It would seem that, unless a student becomes actively involved in a developmental class in college, there is a strong possibility that the student will drop out when outside work or pressures make demands. They often feel, "It's a non-credit course, anyway. It doesn't count." This is a very difficult attitude to change. If we once can involve the students in the course, they usually try to finish.

Tests seem to be the big hassle for students. From the first day, then, we keep this test phobia in mind and try to get the students involved to such a point that this testing is a natural step in the involvement procedure. Beginning with professor-initiated work, we have moved to having students provide steps in the work from their seats, to asking a student to write the steps at the board, while others at their seats dictate the steps in the working of a review problem. The professor, meanwhile, has taken a seat at the back of the room, putting the focus on student involvement. Of course, the instructor steps in and helps. Having one student write the one problem at a time seems preferable to sending many students at once, sometimes very confusing. Also, the entire class is focusing its attention to the one step of the one problem being worked.

7. Outside Help

Students are encouraged to seek outside help in

(a) My office - We are required to be in the office 5 hours per week, and I give help there on a one-to-one basis.

(b) The Learning Center - Students can get free tutoring there on a regular or walk-in basis from a professional tutor, or a paraprofessional, or a peer tutor.

(c) Videotapes - Several years ago I made some videotapes for my students on multiplication, factoring, and other tapes. However, more of the tapes I made were for Intermediate Algebra.

(d) A Second Session - A student is welcome to sit in on a second session of the class, as long as it doesn't disturb or hold back the other section.

(e) Review Sessions - In our free time, if enough students request it, we schedule a review session before a test or exam, if a room is available. Students take turns writing at the board, while the instructor sits with the others.

As mentioned before, one of the main objectives of this type of activity is to shift the involvement from instructor to student, till the student does not feel threatened so much by working the problems on test day.

8. Check Tests

The students know to be prepared to expect an unannounced pop quiz, or check test, during the first five minutes or so of class, on work covered the previous lesson. The students' attendance and promptness improve, as they realize these check tests can not be made up. The first check test of a semester is a practice test, with problems which students can do easily, to inspire

confidence and to dispel fear in the students.

9. Cumulative Grades

Though it may seem that we emphasize tests and grades, we try not to give undue importance to them. But they are important to the students.

Often, all a student needs to motivate him/her to do better is to see where he/she is now, as to grades, and to see how close that is to a good grade.

Then, when I have time, several times a semester, I calculate the students' average on check tests and tests and hand the averages to them personally. This provides a strong motivating factor for some and a clear warning to others. Students will put on the steam when studying for a test if they know they are one point away from the grade they want.

10. Tests and Exams

In my Fundamentals of Algebra classes we have tests on:

Signed Numbers
Polynomials
Products and Factors
Linear Equations
Fractions

and Exams :

Mid-Term Exam (50% of Mid-Term Grade)
and Final Exam (50% of Final Grade).

I tell my students that the Mid-term and Final examinations are worth 50% of the grade in order to give a student another chance to do correctly a problem missed on a test. Really, what is important is that, by the time the student finishes a course, he or she should be able to do the work in the syllabus. Still, work done in the semester counts. Of course, every instructor has

worked out a system which best suits the needs of his/her students.

For security purposes, it seems necessary for me to give 3 different tests to Fundamentals of Algebra students in 12:00, 2:00, and 3:00 sections.

For review purposes, I offer to my students any old tests and solutions from my files. If I were teaching Calculus, maybe I would not do this. But, it seems to me that in Fundamentals of Algebra, we are trying to do all we can ethically do to bring up to standard the Mathematics proficiency of a student who has taken the same course at least once before. No matter how the form of the test changes, there are going to be the same basic skills and types of problems on all of them. So the students are welcome to pick up and use these tests from a week or two before their actual test. Again, the old test provides them with an idea of the whole scope of the coming test, something to shoot for, something to prepare for, something not to be afraid of.

The Test: Perhaps insignificant features are

a. Page 1 - Contains directions and a section for answers. Though I look at the whole test, I want the student to make a conscious decision of what the answer should be. Sometimes a student, unsure, will give a choice of 2 answers.

b. Extra Page 1 - The student may copy his/her answers on the extra Page 1 and take it from the test. After 40 minutes the answers are posted outside - not earlier, as they will rush. Knowing the answers will be posted motivates the students to try to do the problems correctly. Instant feedback helps them to retain the correct method, it seems.

c. Solutions to the Test - When the tests are returned to the students, I try, whenever possible, to give each one of them a copy of the solutions. This is easy, given our excellent Production Department. All that is necessary is to take a copy of the test, work the problems, and Production uses their offset printing method to duplicate as many copies as necessary.

If most of the students missed a certain test or a certain problem, we then spend class time on it. However, usually it is enough for the student to see the correct work. If a student needs more help than that, help in the office seems better.

Obviously, once solutions are distributed, the test cannot be used again. This necessitates composing new tests each semester, which is very time-consuming. But the benefits outweigh the disadvantages; it makes for a very long work week, which many of our Department would readily admit. One drawback to making solutions available is that, reluctantly, I had to say that there would be no makeup. The practice of makeup tests is a much-abused option in some colleges, it would seem. Most students seem to find a way to make it to tests. Still, I have problems with this, even though I readily admit the advantages of making test solutions available.

In checking tests, I try to find some word to encourage and/or challenge the student. It is hard for us who teach in the developmental area to realize that what we say may

seem important to the student. Perhaps that feeling comes from the turbulent past years in college--times in which all authority was generally resented. Also, many, not all, of our students have very low self-esteem, or are society's dropouts or victims, or both. Students may not realize till later that something we have said or done has made an impact on them. I remember a Viet Nam veteran who struggled just to get to class but who eventually overcame his fear of Mathematics and became a strong student. I did not realize till after he had died in an apartment fire and I talked to his parents, that he had been struggling with a serious drug problem, and the notes and words of encouragement on his Mathematics tests and check tests had served as a motivating force when he had trouble making it from one day to the next. Among his things were all of his papers from his Fundamentals of Algebra class. Every instructor here probably has many students who may seem not to be listening but who are taking in and thinking over what you have said or done.

11. Faculty Self-esteem, Appreciation, and Support.

All teaching is difficult today. But teaching any developmental course has its special problems, many of which originate outside the classroom. First, we are teaching ninth grade Algebra to college students, some of whom feel humiliated to be taking "Mickey Mouse Math." In the area of developmental reading, the educators have recognized this to be a problem and haven't served up Dick and Jane to older students with reading problems. To

adapt the material, approach and format to the variety of students who have developmental algebra needs is a most difficult and frustrating endeavor. On the other hand, the sequential nature of Mathematics requires that we cover all the required materials, if students are to go on to the next course. Also, Mathematics is one of those subjects which require the student to keep up to date on assignments. Students are critical, saying, "I'm making a 4.0 in my other subjects, and this is the only one I'm having any trouble with." OR " I made a B in high school algebra; I don't know why I have to work so hard now." It is difficult to answer such comments candidly, as many factors have contributed to the altered present situation. These and other factors contribute to the low esteem in which some developmental Algebra instructors in college view themselves. Add to that the public's general rejection of the idea that any instructor teaching 15-20 hours in a classroom could possibly be putting in a 50-60 or 70-hour work week! Add to that the devastating effect of some students' comments when they evaluate professors.

It is not difficult to understand that "faculty burnout" is a phrase which has come into more use. Rather than to push over the hill those of us who have labored long and wearily, foresighted administrators lend support and encouragement to these people, who are often some of the college's most valuable assets.

I have to remind myself that students do not appreciate an overworked, cranky instructor. They do not want to know my problems.

What keeps a good teacher coming back for more developmental Algebra? One of the most potent motivating forces is other supportive faculty and administrators. Find and cultivate appreciative

co-workers and administrators. You need one another. A problem seems less threatening when you find you are not alone. A word of credit--not a phony pat on the back--but a genuinely appreciative word of support to a colleague goes a long way. That, coupled with the student who confides, "I never got Math before, and I am now," and the graduate who returns after passing Differential Equations, and says, "Thanks for making me work"--all of that may be part of the reason we still have moments of exhilaration in teaching fundamental Mathematics.

These steps I have outlined above seem to help students to master Fundamentals of Algebra and to make a smoother transition to the next course. However, they are not the only way, and I try to keep changing to adapt to the students' needs.

I offer the following words I use in part to introduce my students to this course:

<u>First, Why Study Math?</u>

Of course, some careers require a specific knowledge of mathematics, so some students need math for that reason.

What about careers that don't call for mathematical skill? Everyone knows there's a lot more to a person's life than a job or career. That's where learning mathematics can help you, we think--in all you do--thinking, planning, organizing throughout your whole life.

As you know, when you study mathematics, you have a statement, or some facts, or an expression, and the problem asks you to do something with it--solve, simplify, or come up with another fact.

In order to do this, you must be able to distinguish the important facts

from the unimportant.

You must plan how you will get from where you are now to where you hope to end up. In order to reach that goal, you must organize your thoughts and your facts.

You have to stick with what you believe is right, even when you are wondering about the answer. And you have to check your answers to see if they are correct.

This planning, thinking, organizing, persevering, and checking, which you learn to do with mathematical problems, will help you to strengthen the same skills in everyday life, with its many problems.

This methodical, careful way of thinking and acting is, I believe, the greatest benefit you can take away from any course in mathematics and a college education.

Both the study of mathematics and a college education should make a person more open-minded, ready to explore all reasonable possibilities to a situation.